Abstract—The mobile location in wireless communication systems becomes a popular issue in recent years. It is commonly used in emergency services, roadside assistance, billing, navigation, psychophysics, behavioral science, etc. Over recent decades, many location algorithms are put forward. However, the algorithms result in inaccurate location estimation due to Non-Line-Of-Sight (NLOS) propagation. This paper proposes the mobile location estimation based on the Received Signal Strength (RSS) and the Time-Of-Arrival (TOA) measurements, via the multidimensional scaling technique (MDS). The algorithm does not require a known and accurate path loss modeling, reduces the impact of NLOS on location. Simulation results show that the algorithm performs well even though in the severe NLOS scenario.

Keywords— mobile location estimation; multidimensional scaling; NLOS; RSS; TOA

I. INTRODUCTION

Mobile locations could support many applications, such as emergency services, roadside assistance, navigation, tracking, and so on [1]. The major driving force is due to the Federal Communications Commission (FCC) put forward the accuracy requirement of Enhanced-911 (E-911) safety services [2]. With the popularization of wireless services, more and more people call for emergency services by wireless phone [3], and the system operators want to provide subscriber’s location information which conforms to the FCC requirement. Therefore, the location estimation has received considerable attention over the past few years.

In the last few years, location estimations could be divided into five categories. The first category is the Global Positioning System (GPS) [4], which provides accurate positioning but fails when satellite signals are blocked, for example the mobile station (MS) in the indoors or urban canyons environments. The second category estimates the MS location based on RSS measurements [5]. This method is employed to measure the power of the signal at the receiver. Relatively, low accuracy is achieved in this way due to the complex propagation mechanisms. The third category uses the angle-of-arrival (AOA) [6]. However, this method requires a complex antenna array system and suffers from NLOS propagation. The fourth category is the TOA [7]. The TOA scheme measures the arrival time of the radio signal coming from the base station (BS). Each TOA defines a circle centered on the BS. The MS position can then be determined at intersection of circles. Similar to AOA technique, NLOS propagation is a potential disadvantage of TOA method. The fifth category identifies the MS location by matching the received signal signatures with the entries stored in a database at the network [8]. The location accuracy depends strongly on channel variations and the size of the database, which is time-consuming for construction and update.

The MDS is a method that represents measurements of similarity (or dissimilarity) among pairs of objects as distances between points of a low-dimensional multidimensional space [9]. It describes the structure of a set of objects from distances between pairs of the objects. Over recent decades, the MDS has been proposed for wireless network based mobile location. However, classical multidimensional scaling is sensitive to the range measurements error in the NLOS scenario [10]. Due to the MDS needs distance measurement data between the MS and BSs, the MDS technique has been proposed for mobile location using the TOA method [11-14]. In [11], a location algorithm is derived via modifying of the MDS. In [13], a location algorithm is derived via weight vector. In [15], an extension of the MDS is given, which uses distance and angle information. However, they are sensitive to the range measurements error due to NLOS propagation.

Each location method has advantage and weaknesses. Most of them require multiple BSs to work. Clearly, adding more extra information is expected to yield smaller errors. Therefore, the hybrid techniques, such as hybrid SADOA/TDOA [16, 17], hybrid TOA/AOA [18] and hybrid TDOA/AOA [19], have been suggested. The hybrid techniques promote location accuracy even though without increasing the number of BSs. However, the hybrid TOA/AOA and hybrid TDOA/AOA can't
provide an accurate location estimation when the NLOS propagation interference very serious.

This paper presents a hybrid multidimensional scaling algorithm for mobile location. It improves the performance of mobile location by combining the RSS and TOA information, via the MDS technique. This method does not require a known and accurate path loss modeling, reduces the impact of shadowing and NLOS on location accuracy.

The rest of this paper is organized as follows. Section II presents the SADOA (signal attenuation difference of arrivals) method, and hybrid MDS schemes. By simulation, Section III discusses the location performances in different shadowings, NLOS propagation environments and abilities to detect BSs. Conclusions are given in Section VI.

II. PROPOSED HYBRID SCHEME

This section gives the proposed SADOA method, reviews the MDS method, and presents the proposed Hybrid-MDS scheme.

A. SADOA method

In a real propagation environment, path loss and shadowing influence the received signal power of reception. The path loss comes from that all of the loss elements associated with interactions between the propagating wave and any objects between the transmitting and receiving antennae. Shadowing results from different clutter environment (buildings, trees, etc.) along the path traveled by the wave, causing signal variations with respect to the nominal value given by path loss models. Shadowing is generally assumed to be a lognormal distributed random variable, which is a zero-mean Gaussian random variable when expressed in decibels [20]. Hence a generalized form for signal attenuation, \( A \), between the BS and MS separated by \( d \) can be expressed as

\[
A(dB) = k_i + k_2 \log_{10} f + k_3 \log_{10} h_i + 10 \log_{10} d + k_4 \log_{10}(kh_i^n) + s
\]

where \( k_i, k_2, k_3, k_4 \), and \( k_5 \) are different constants in the same clutter type of environment, \( f \) is the carrier frequency, \( h_i \) and \( h_j \) are, respectively, the height of BS and MS, \( n \) is the path loss exponent, and \( s \) represents a zero-mean Gaussian random variable with respect to the nominal value given by path loss models.

In the two-dimension scenario, the location of MS is estimated by connecting with \( N \) BSs denoted as \( BS_i \) and located at \((x_i, y_i), i = 1 \ldots N\). Assume that 1) all BSs operate at the same frequency band; 2) the all BSs have same height; 3) the all links are in the same type of clutter environment. Consequently, the parameter sets, \( \{k_i, k_2, k_3, k_4, k_5, n\} \), are identical, then the difference between the attenuations of signals from \( BS_i \) and \( BS_j \) can be denoted as

\[
A_i - A_j = 10n \log \frac{d_i}{d_j} + (S_i - S_j)
\]

where the \( d_i \) and \( d_j \) are the distances between the MS and two BSs (\( BS_i \) and \( BS_j \)), then \( S_i \) and \( S_j \) are the shadowing of the propagation paths. Therefore, it can be derived the ratio of \( d_i \) to \( d_j \), symbolized by \( p_{ij} \), from (2),

\[
p_{ij} = \frac{d_i}{d_j} = 10^{\frac{A_i - A_j}{10n}} - S_i
\]

where \( S = (S_i - S_j)/10n \) is a zero-mean Gaussian random variable. Denote the standard deviation (STD) of shadowing and correlation of shadowing as \( \sigma_s \) and \( \alpha_s \), respectively, then the variance of \( S \) is \( 2\sigma_s^2(1-\alpha_s)/(10n)^2 \). The ratio can determine the relation between the MS and two BSs, which is express as

\[
p_{ij} = \frac{(x-x_i)^2 + (y-y_i)^2}{(x-x_j)^2 + (y-y_j)^2}
\]

after advisable arranging, the (5) can be rewritten as

\[
(x-x_{ij})^2 + (y-y_{ij})^2 = R_{ij}^2
\]

where \( x_{ij} = (p_{ij}x_i - x_j)/(p_{ij}^2 - 1) \) and \( y_{ij} = (p_{ij}y_i - y_j)/(p_{ij}^2 - 1) \), and \( R_{ij} = |p_{ij}^2 - 1|^{-1} \).\( D_{ij} \) is the distance between \( BS_i \) and \( BS_j \). The SADOA method locates the MS position with a geometrical approach, which generates linear lines of position (LOP) by differencing pair of circular and uses the least-squares (LS) algorithm to solve the MS position. For simplicity, set \( N = 3 \) and LOP determined by two circles, centered at \((x_i, y_i)\) and \((x_j, y_j)\) and with radiuses of \( R_i \) and \( R_j \), where \( i, j = 1 \ldots M \) \( M = N(N-1)/2 \). Then, express the set of linear LOPs in matrix form, we have

\[
AX_{MS} = B
\]

where

\[
A = \begin{bmatrix}
 x_{i2} - x_{i1} & y_{i2} - y_{i1} \\
 x_{i3} - x_{i1} & y_{i3} - y_{i1} \\
 x_{j2} - x_{j1} & y_{j2} - y_{j1}
\end{bmatrix}, X_{MS} = \begin{bmatrix} x \\ y \end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
 R_{i1}^2 - R_{i2}^2 - (x_{i1}^2 + y_{i1}^2) + (x_{i2}^2 + y_{i2}^2) \\
 R_{i1}^2 - R_{i3}^2 - (x_{i1}^2 + y_{i1}^2) + (x_{i3}^2 + y_{i3}^2) \\
 R_{j1}^2 - R_{j2}^2 - (x_{j1}^2 + y_{j1}^2) + (x_{j2}^2 + y_{j2}^2)
\end{bmatrix},
\]
The LS solution is derived from

\[ X_{\text{LS}} = \left( A^T A \right)^{-1} A^T B \]  

(8)

B. Classical MDS

It is common to use classical MDS with TOA measurement [11]. When measuring the TOA data, the measurements can be modeled as

\[ r_i = d_i + n_i \]  

(9)

where \( r_i \) is the range measurement between the MS and \( B_i \) and the \( n_i \) is the range error. The error was resulted from multipath and NLOS propagation, which both cause the time delay effect, so we model the error as zero-mean and single-side Gaussian random variable.

As described in [10], when applying classical MDS to estimate mobile location, it is primary to construct a matrix of squared distances between pairs of BSs and MS, as

\[
G = \begin{bmatrix} 0 & h_i \\ h_i^T & D_i \end{bmatrix}
\]  

(10)

where

\[
D_i = \begin{bmatrix} d_{i1}^2 & \cdots & d_{iN}^2 \\ \vdots & \ddots & \vdots \\ d_{i1}^2 & \cdots & d_{iN}^2 \end{bmatrix}
\]  

(11)

and

\[
h_i = \begin{bmatrix} r_i^2 \\ r_i^2 \\ \vdots \\
\vdots \\ r_i^2 \end{bmatrix}
\]  

(12)

where \( d_{ij} \) is the distance between the \( BS_i \) and \( BS_j \) with free of error, and the \( r_i \) is the range measurement between the MS and \( BS_i \), then compute the matrix \( D_i \) with double centering to obtain a scalar product matrix, which is denoted by \( B_i \), and is of the form

\[
B_i = -\frac{1}{2} J_{N+1} G J_{N+1}^T
\]  

(13)

where \( J_{N+1} = I_{N+1} - (1/N+1) \cdot 1_{N+1} 1_{N+1}^T \) is the centering matrix, with \( I_{N+1} \) and \( 1_{N+1} \) denoting the \((N+1) \times (N+1)\) identity matrix and \((N+1) \times 1\) column vector of all ones, respectively. When employing the classical MDS, the centroid of the all coordinates is assumed at the origin, so the scalar matrix, \( B_i \), can be express as

\[
B_i = XX^T = \begin{bmatrix} X_{\text{MS}} \\ X_{\text{MS}} \end{bmatrix} \begin{bmatrix} X_{\text{MS}}^T \\ X_{\text{MS}} \end{bmatrix}
\]  

(14)

where \( X_{\text{MS}} = \begin{bmatrix} x_{i1}^T & x_{i2}^T & \cdots & x_{iN}^T \end{bmatrix} \) denoting the coordinate of \( BS_i \). From (14), it is implying that \( B_i \) is symmetric and the rank of \( B_i \) is 2, hence we can decompose \( B_i \) by using eigenvalue decomposition, it can be shown that

\[
B_i = U \Lambda U^T
\]  

(15)

where \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N) \) is the diagonal matrix of eigenvalues of \( B_i \) with \( \lambda_{\text{min}} \geq \lambda_1 \geq \cdots \geq \lambda_N \geq 0 \), and \( U = [v_1 \ v_2 \ \ldots \ v_N] \) is an matrix whose columns are the corresponding eigenvectors. Since rank of \( B_i \) is 2, we have

\[
X = U \sqrt{\Lambda} \]  

(16)

where \( U = [v_1 \ v_2] \) and \( \sqrt{\Lambda} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}) \). From (14) and (16), we can obtain the coordinate of BSs and MS, which will be relative to origin.

C. Hybrid SADOA/TOA with MDS

The proposed hybrid SADOA/TOA location scheme applies MDS to estimate the MS position based on the RSS and the TOA measurement. It is different from classical MDS the centroid is assumed at the MS position [10]. According to SADOA method, a circle is obtained by the distance ratio, which is determined by signal attenuation. To applying MDS, we consider, separately, the center coordinate and the radius as the virtual BS position and the distance between the virtual BS and MS. With \( N \) BSs, we can obtain two matrixes, which one is denoted as

\[
X_e = \begin{bmatrix} X_{e1}^T & X_{e2}^T & \cdots & X_{en}^T \end{bmatrix}^T
\]  

(17)

and the other is

\[
h_e = \begin{bmatrix} R_{e1}^2 \\ R_{e2}^2 \\ \vdots \\ R_{en}^2 \end{bmatrix}
\]  

(18)

where \( X_{ei} = [x_{ei} \ y_{ei}] \) and \( R_i \) are, respectively, the coordinate of virtual BS and the distance between the virtual BS and MS. With \( N \) BSs, we can obtain two matrixes, which are expressed as

\[
D_e = \begin{bmatrix} D_e \\ D_e \\ \vdots \\ D_e \end{bmatrix}
\]  

(19)

and

\[
h_e = \begin{bmatrix} h_e^T \\ h_e^T \\ \vdots \\ h_e^T \end{bmatrix}
\]  

(20)

where

\[
\begin{bmatrix} d_{e1}^2 & d_{e2}^2 & \cdots & d_{en}^2 \\ d_{e2}^2 & d_{e2}^2 & \cdots & d_{en}^2 \\ \vdots & \vdots & \ddots & \vdots \\ d_{en}^2 & d_{en}^2 & \cdots & d_{en}^2 \end{bmatrix}
\]
\[
D_x = \begin{bmatrix}
0 & d_{12}^2 & \cdots & d_{1N}^2 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & d_{N2}^2 \\
d_{N1}^2 & d_{N1}^2 & \cdots & 0 \\
\end{bmatrix},
\]

with \(d_{ij}\) and \(d_{ij}(i \neq j)\) denoting as the distance between the BS\(_i\) and virtual BS\(_j\), and the distance between the virtual BS\(_i\) and virtual BS\(_j\), respectively. From cosine theorem [10], use the (19) and (20) to obtain a scalar product matrix, which is expressed as
\[
B_x = X_h X_h^T = \frac{1}{2}(h_b^T 1 + 1 L - h_b^T - D_x) \tag{21}
\]

where the \(1_L\) is the \(L \times 1\) column vector and \(L = N + M\). Since the centroid is assumed at the MS before, \(h_b\) can also be represented by
\[
B_x = X_h X_h^T = \begin{bmatrix}
X_1 & X_2 & \cdots & X_N \\
\end{bmatrix}
\begin{bmatrix}
X_1^T \\
X_2^T \\
\vdots \\
X_N^T \\
\end{bmatrix} = \begin{bmatrix}
X_1 - X_{MS} \\
X_2 - X_{MS} \\
\vdots \\
X_N - X_{MS} \\
\end{bmatrix} \tag{22}
\]

where \(X_i = X_{b2} - 1_L X_{MS}\)

and
\[
X_i = X_{ci} - 1_M X_{MS} = \begin{bmatrix}
X_{c1} - X_{MS} \\
X_{c2} - X_{MS} \\
\vdots \\
X_{cM} - X_{MS} \\
\end{bmatrix} \tag{23}
\]

Since (22) is similar to (14), the \(B_x\) is symmetric and the rank of \(B_x\) is 2, so it is able to decompose \(B_x\) with eigenvalue decomposition. Then \(B_x\) can also be shown that
\[
B_x = U_x \Lambda_x U_x^T \tag{24}
\]

where \(\Lambda_x = \text{diag}(\lambda_{x1}, \lambda_{x2}, \ldots, \lambda_{xL})\) is the diagonal matrix of eigenvalues of \(B_x\) with \(\lambda_{x1} \geq \lambda_{x2} \geq \ldots \geq \lambda_{xL} \geq 0\), and \(U_x = [v_{x1} \ v_{x2} \ \ldots \ v_{xL}]\) is an matrix whose columns are the corresponding eigenvectors. Since rank of \(B_x\) is 2, we have
\[
\lambda_{x1} \approx \lambda_{x2} \approx \ldots \approx \lambda_{xL} \approx 0 \tag{25}
\]

From (22), (23) and (24), we can obtain
\[
\bar{U}_x \Lambda_x \bar{U}_x^T = 0_{L-3p2} \tag{26}
\]

Where \(0_{(L-3)p2}\) is matrix consisting entirely of zeros, and \(\bar{U}_x = [v_{x3} \ v_{x4} \ \ldots \ v_{xL}]\). According to (22), the \(X_h\) can be expressed as

\[
X_h = X_{MS} - 1_L X_{MS} \tag{27}
\]

where \(X_{MS} = [X_{b1} X_{b2}^T]^T\). Substituting (26) into (25) and arrange it, we can get
\[
\tilde{U}_x^T 1_L X_{MS} = \bar{U}_x^T X_{MS} \tag{28}
\]

The MS position is estimated by LS method, it is derived from

\[
\hat{X}_{MS} = \tilde{U}_x^T 1_L \tilde{U}_x^T 1_L \tilde{U}_x^T X_{MS} \tag{29}
\]

### III. LOCATION ESTIMATION SIMULATION

In this section, using MATLAB simulation results to corroborate the theoretical development, Fig. 1 illustrates the hexagonal tested cell surrounded by six neighboring cells with radius of 250m. The Cost231-Hata model and a Gaussian random variable were applied for the path loss and shadowing simulations, respectively. The Gaussian random variable was provided for the NLOS. 10,000 MSs are uniformly distributed in the center cell.

Xia et al. found that the STD of shadowing ranges from 4.2 to 7.7dB in residential/suburban environments and from 2.2 to 8.3dB in urban environments for microcells operating in the 900MHz frequency band [21]. In [22], it found that the STD of NLOS = 250m in the execrable real NLOS cases. Moreover, Saunders found that the correlation of shadowing ranges from 0.3 to 0.8 when \(d_1\) is 1 km and \(d_2\) is 2 km [23]. Fig. 2 illustrates the location performance with different correlation of shadowing and five BSs per estimate with the STD of NLOS = 250m. The ordinate is the location errors, which represents the performance of 67% of the estimates, and abscissa is the STD of shadowing. The curves with different shapes are performances with different shadowing correlation coefficients.
In universal mobile telecommunications system terrestrial radio access networks (UTRAN), the impact of ability to detect BSs on location estimation should be explored. Fig. 3 illustrate the performances of the proposed method with different abilities to detect BSs and the STD of shadowing with the correlation of shadowing = 0.4 and the STD of NLOS=250m. The location errors are below 122.19m with the STD of shadowing = 4 dB, and rise to 159.14m with the STD of shadowing = 8 dB with five BSs. Fig. 4 illustrates the performances of the proposed method with different abilities to detect BSs and the correlation of shadowing with the STD of shadowing = 8 dB and the STD of NLOS=250m case. The location errors are below 131.45m with the correlation of shadowing = 0.8, and rise to 169.4m with uncorrelated shadowings with five BSs. Fig. 5 illustrates the performances of the proposed method with different abilities to detect BSs and STD of NLOS with the STD of shadowing = 8 dB and the correlation of shadowing = 0.4.

Additionally, the proposed method is compared with the Cell-ID method, the TDOA (Time Difference of Arrival) based on Taylor-series (TS) method, the Classical MDS, and the SADOA -LOP method.

Table I summarizes the statistics of the location estimation simulations. The cell layout in the case is the same as in previous simulations. The path loss exponent is set to 3.46, and the STD of NLOS is set to 250m. To carefully verify the performance of the proposed method in fading environments,
the STD of shadowing and the correlation of shadowing are respectively set to 5.5-dB and 0 in the simulations. Compare the classical MDS method and the proposed method, 67% of the location errors are below 238.84m in the classical MDS case. Using the proposed method, the error is 153.75m, representing 35.7% improvement compared to the classical MDS method and 28.8% improvement compared to the Cell-ID method. Compare the Classical MDS, the SADOA-LOP and the TDOA method, the location error of the Hybrid SADOA/TOA-MDS is more stable.

<table>
<thead>
<tr>
<th></th>
<th>LOCATION ERROR(METERS)</th>
<th>Mean</th>
<th>Std</th>
<th>67%</th>
<th>95%</th>
</tr>
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<tbody>
<tr>
<td>Cell-ID</td>
<td>201.98</td>
<td>138.78</td>
<td>215.79</td>
<td>246.46</td>
<td></td>
</tr>
<tr>
<td>TDOA (IG: Serving BS)</td>
<td>182.62</td>
<td>325.39</td>
<td>153.92</td>
<td>763.21</td>
<td></td>
</tr>
<tr>
<td>SADOA-LOP</td>
<td>169.62</td>
<td>155.9</td>
<td>161.31</td>
<td>422.12</td>
<td></td>
</tr>
<tr>
<td>Classical MDS</td>
<td>212.7</td>
<td>166.23</td>
<td>238.84</td>
<td>476.61</td>
<td></td>
</tr>
<tr>
<td>Hybrid SADOA/TOA-MDS</td>
<td>151.9</td>
<td>149.58</td>
<td>153.75</td>
<td>252.23</td>
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<tr>
<td>Improvement over Cell-ID Method at the 67%</td>
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<td></td>
<td>28.8%</td>
<td></td>
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<tr>
<td>Improvement over SADOA-LOP Method at the 67%</td>
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<td></td>
<td>4.7%</td>
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<tr>
<td>Improvement over Classical MDS Method at the 67%</td>
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<td></td>
<td>35.7%</td>
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</tr>
</tbody>
</table>

Table I. Statistics of Location Estimation Errors

IV. CONCLUSIONS

This paper proposes a hybrid SADOA/TOA location scheme for wireless communication systems. By introducing a linear form of the SADOA equation and MDS algorithm, we extend the previous TOA only location estimators to solve the SADOA/TOA equations for the 2-D array BS layout case. The SADOA algorithm provides virtual distance between MS and BS, which does not require a known and accurate path loss modeling, reduces NLOS impact on location, and can be applied to an existing system without hardware modifications. Even though only few BSs are available, the proposed scheme still supports good location performance. Simulations demonstrate that the proposed scheme generally performs better than the Classical MDS and the Cell-ID in the NLOS scenario.

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REFERENCE